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Jiang Yang is currently full Professor in the Department of Mathematics at Southern University of Science and Technology (SUSTech). He received his Bachelor's degree from Zhejiang University in 2010 and his PhD from Hong Kong Baptist University in 2014. Prior to joining SUSTech in 2017, he conducted postdoctoral research at Pennsylvania State University and Columbia University. His research focuses on computational mathematics, with primary interests including the modeling, numerical methods and applications of phase-field models and nonlocal models, as well as the design and theory of deep learning algorithms. His work has been honored with EASIAM Student Paper 2nd Prize at the East Asian SIAM Conference (EASIAM 2014) and the "Frontier Science Award" at the International Congress of Basic Sciences (ICBS 2025).

Title: High-order structure-preserving product-type runge–kutta methods for gradient flow models with variable mobility

Abstract:
Gradient Flow Models serve as essential mathematical tools for describing evolutionary laws in nature, with extensive applications in the heat equation, phase-field equations, Ricci flow, surface minimization problems, and stochastic gradient descent algorithms in machine learning. In numerical simulations, the rigorous preservation of the original energy dissipation law remains a core challenge in this field. While classical algorithms—such as stabilization methods, convex splitting, invariant energy quadraturization methods, scalar auxiliary variable methods, and implicit-explicit/exponentialtime-differencing Runge–Kutta methods—have achieved significant success for constant-coefficient models, existing algorithms face limitations in balancing structure preservation and computational efficiency when dealing with nonlinear variable mobility. To address these challenges, this report proposes a suite of innovative high-order Product-type Runge–Kutta (P-RK) methods. By investigating three representative nonlinear models, we demonstrate the superior performance and flexibility of the proposed methods in structure preservation:

- Dirichlet Harmonic Map Gradient Flow: By constructing appropriate P-RK methods combined with a normalization post-processing step, we achieve dual structure preservation of the energy dissipation law and the unit-length constraint ($|\mathbf{u}| = 1$). To the best of our knowledge, this is the first second-order scheme that maintains both structures while maintaining linear computational complexity.
- Anisotropic Dendritic Growth Phase-field Model: By introducing P-RK methods with multi-block Butcher tableaus, we develop an efficient decoupled algorithm that strictly follows the original energy dissipation structure while significantly enhancing computational efficiency.
- Variable-mobility Allen–Cahn Equation: By constructing appropriate P-RK methods combined with a cut-off post-processing technique, we achieve unconditional preservation of both the original energy dissipation and the Maximum Bound Principle (MBP). To overcome the difficulties in optimal error estimation caused by truncation post-processing, we propose a time two-grid method, which successfully restores the optimal convergence order.

Both theoretical analysis and numerical experiments demonstrate that the proposed methods possess significant advantages in structure-preserving properties, numerical stability, and computational accuracy. This work provides a new theoretical framework and numerical tools for the efficient and high-performance simulation of complex gradient flow systems.